OPTIMISATION OF LOCATING PROBLEMS IN URBAN CONTEXTS

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ABSTRACT: The object of the paper is the analysis of a location-like problem with reference to an urban context.

The uncapacitated and capacitated urban functions and facilities are considered. This kind of problem is usually stated as follows: given m clients and n potential sites for locating prespecified functions and facilities select an optimal set of locations.

The paper formulates a traffic network model oriented to the analysis, simulation and sub-optimal solution to a location-like problem. The model is analogue as it describes the road network as an electric network; it describes each lane of a road as an oriented link, characterised by (time-varying) parameters, such as the route time at given unsaturated conditions. Road intersections are represented as nodes with inflows and outflows. The effect of the location of a facility or a function in a precise site over the surrounding extended area is particularly modelled.

The proposed model is based on an interactive software package, as a decision support system, supported by a graphical front-end named ULISSE, on LINUX platform, which shows the plan of the selected city, and that permits to show the result of the simulation with the possibility to change each parameter of the net directly on a friendly graphical front end.

The main result is the optimised location of the facilities or functions in the urban context, visualised directly on the city plan. Other results are: -the influence area of each service site; -the degree of utilisation of each service; -the induced traffic flow increment on each road; -the total cost of any given service location. Moreover other results can be obtained: -the minimum path matrix for all the nodes of the network; -the distribution of traffic due to a given origin-destination matrix.

KEY WORDS: Locating Problems, Location Support System, Mobility Model, Interactive Optimisation, Graphical Front-end

1 Introduction

When approaching to territorial problems it's basic to control two different layers, the data and model bases, and the real representation. The new software offers great possibilities of management of each step of the process of knowledge, analysis, interpretation and simulation of the phenomena. Urban Planning needs to be closely connected with both of these layers; the urban models are well structured since 1950 but the difficult in obtaining and treating a great quantity of different kind of data got always to untopical forecasts. This kind of problem is now solved with the actual calculation power of simple PC while the representation of future settings is still growing up. Moreover each specific problem needs specific models and representations.

One of the main problems, in urban planning, is the location of the different urban function (residence, services, productive, etc.) trying to find the optimal solution. Actually great importance is given to facilities.

To formulate a facility location problem with reference to a particular urban context numerous factors need to be taken into account. Nevertheless, it is worth considering that, in urban planning, more than an optimal solution, a suboptimal feasible solution to the location problem, iteratively constructed through the analysis of different aspects, such as the travel times to reach a service, the traffic increment, and the environmental impact, can be preferable. In both cases, a classical way to face the problem is to define a global cost function associated with a certain location decision to be minimised.

The cost function can rely, for instance, on the actual cost to build and run the facility together with the costs paid by the facility clients. But while the first type of cost can be reasonably regarded as independent of the site chosen for location, the costs paid by the facility clients determine a distribution of the potential users on the urban area, significantly affecting the traffic flows through the underlying transportation networks. These costs can be quantified in terms of the time to reach and access the facility in question. So, for the sake of simplicity, one can begin by assuming that the user selects the facility, which turns out to be "closer" in a temporal sense (i.e., with associated minimum access time).

Making reference to a real urban context the concept of “client” can be naturally replaced with the concept of “vehicle” since it is impossible to ignore the presence of an underlying transportation network through which the *i*-th client reaches the *j*-th facility. According to this basic consideration, this paper formulates a traffic network model oriented to the analysis, simulation and sub-optimal solution to a location-like problem. The proposed model describes the road network as an electric network, enabling to evaluate the incremental traffic due to the access to the service. It describes each lane of a road as an oriented link, characterised by (time-varying) parameters, such as the travel time at given unsaturated conditions. Road intersections are represented as nodes with inflows and outflows. The propagation delays due to the presence of traffic lights and non-homogeneous flows are also taken into account. Finally the effect of the location of a facility in a precise site over the surrounding extended area is modelled.

The proposed model is the basis of an interactive software tool, ULISSE (Urban Location Interactive System for SErvices), that can be used as a decision support system, for instance, by Urban Planning Experts or Public Administrators. This system allows to have a friendly interface based on the plan of the considered city, whether a simple map or a GIS based system of thematic maps in different layers, and to operate interactively following each step of the research of the optimal solution.

2 Urban context and location problems

The optimal location of facilities in an urban context depends on numerous factors. A classical way to face the problem is to define a global cost function associated with a certain location decision to be minimised. The cost function can rely, for instance, on the actual cost to build and on the facility together with the costs paid by the facility clients. The first type of cost can be reasonably regarded as independent of the site chosen for location, that is, it is not so relevant in the minimisation process.

In contrast, the costs paid by the facility clients determine a distribution of the potential users on the territory; significantly influencing the traffic flows through the underlying transportation networks. These costs can be quantified in terms of the time to reach and access the facility in question. So, for the sake of simplicity, one can assume that the user selects the facility, which turns out to be “c1oser” in a temporal sense (i.e., with associated minimum access time). Also the type of transportation mean used by each client has an influence on the global cost function. Thus, assume to subdivide the considered urban region into N areas, to locate the facilities in certain precise positions, and to account for L possible types of transportation means. Then by analogy with [1], a possible way to express the cost to be minimised is the following:

$$c = \sum_k \sum_j \sum_i p_i^k c_i^k t_{ij}^k \tag{1}$$

where p^k_i is the user population in the i^{th} area using the means of transportation *k,* c_i^k is the weighting factor associated with that type of means of transportation, t^k_{ij} is the time to reach from the i^{th} area, the j^{th} facility with the means of transportation *k.* The solution to the complex problem of the minimisation of cost (1) by suitably locating the facilities is within the scope of operational research. Yet, in practical planning situations, more than an optimal solution, it is sought an instrument supporting urban planners throughout the location decision process. This instrument should quickly provide, for instance, for any location trial suggested by the territorial designers, such useful indications as the value of the cost *C,* the influence areas of each facility, as well as its saturation level, the increment of induced traffic on each links of the underlying transportation network.

3 Defining a model

The design of a planning decision support instrument cannot proceed without the formulation of a mathematical model of the access to each facility. The model here proposed relies on the assumption that the dominant transportation mode (*dominant* in the sense that it primarily contributes to determine the access time) is the private vehicle.

3.1. Formulation

Consider the problem relevant to the facilities *Sj,* 1<j<M, of a certain nature to be located in an urban context under the following assumptions:

1. each facility has the assigned capacity to serve *Cj* clients per hour;

2. the distribution of the potential clients over the urban territory is known (more precisely, the spatial distribution is discretized into elementary units called “cells");

3. each cell *i*, 1<*i*< N, contains $p_i(t)$ potential clients, where such a quantity is a time function valued in number of clients per hour;

4. the i^{th} cell is centered in the i^{th} road intersection 1<*i*<N, N being the total number of road intersections of the urban transportation network considered.

The p_i-th client's choice of the facility to reach is assumed to be dictated by the cost to access the facility. In the case of transportation by means of private vehicles, such a cost can be modelled as directly proportional to the vehicle travel time t(i,j) from the i^{th} road intersection, from which the p_i-th client starts, to the j^{th} road intersection, where the facility is located. The global vehicle travel time is obviously the sum of the travel times associated with the links of the transportation network through which the client moves to reach the facility.

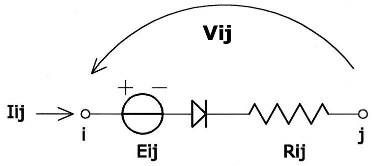
The urban transportation network can be represented by a graph with N nodes and a set of oriented links l(i,j), i<*N,* j<N, connecting the i^{th} node with the j^{th} node, with the travel direction from *i* to *j*. Each link is marked by a label which defines the time-varying travel time law on the link itself as a function of the traffic volume $n_{ij}(t)$.To further refine the model of the access to a facility it is necessary to consider that, relying on the model, the following aspects should be determinable:

1. for any node, the time to access the nearest facility;
2. the nodes “captured” by a facility, and the corresponding burden in terms of clients. This, in turn, allows one to identify the influence area of each facility delimited, on the nodes map, by the borderlines connecting the nodes with associated longer access time
3. the number of clients reaching each facility;
4. the induced traffic variation in each link of the transportation network;
5. the total cost for the users which is implied by the selected facility location: this quantity can be computed by summing up all the access times associated with the nodes multiplied for the number of clients arriving from each node.

3.2 The access time computation

To determine the quantities indicated above, it is necessary to compute the travel time corresponding to each link of the considered transportation network, making reference to the particular traffic conditions in the time interval of interest. The travel time as a function of the traffic intensity, of the parameters which determine the traffic fluidity, and the possible presence of traffic lights is provided. Yet, in our case, the point is to evaluate the traffic variation due to the location of a certain facility in a certain area. Then, the mentioned function can be linearly approximated by the tangent in the operation point in question. More precisely, given the regular traffic in the considered link, one can determine the corresponding travel time $t_0\,(i,j)$.

Figure 1: The electric equivalent of a link of the transportation network



with: $Rij = \left.\dfrac{dt(i,j)}{dn(i,j)}\right|_0$ $\qquad t(i,j) = Vij \qquad t_0(i,j) = Eij \qquad \Delta n(i,j) = \Delta Iij$

Then, the travel time after that the facility has been located can be written as

$$t(i,j) = t_0\,(i,j) + R[n(i,j) - n_0\,(i,j)] = t_0\,(i,j) + R_{ij}\Delta n(i,j) \qquad (2)$$

where $\Delta n(i,j)$ is the traffic intensity induced by the considered facility.

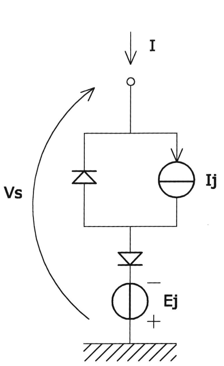
Clearly, in searching the optimal facility location, it is the average access time to be crucial rather than the access time of a single client. This is the reason why a macroscopic continuous-time model turns out to be the correct choice. The role of the voltage generator Eij is to polarise the diode thus representing the original travel time $t_o(i,j)$. The value of the resistor Rij describes the dependence on the current Iij, i.e., on the induced traffic intensity $\Delta n(i,j)$. Finally, the voltage Vij represents the link travel time t(i,j).

Relying on the electrical modelling equivalent, the clients population can be modelled by means of ideal current generators which injects their current in the nodes of the transportation network where the barycenter of each cell is located. The values of the (clients) current are expressed in terms of the number of vehicles per hour (veh/h). They account for both the time distribution and the anagraphical and socio-economic features of the clients population, this through a parameter related to the "appeal" of each facility. Note that with the term facility "appeal", we mean the capability that a certain type of facility has to attract the clients population. Data relevant to this capability can be acquired from national statistical studies centres (for instance, CENSIS). Generally, the available data provide the number of families attracted by the considered facility type, the number of persons for family unit, their distribution over the territory.

From the modelling point of view, each facility, regarded as uncapacitated, is described by setting at a null potential the corresponding node where it is assumed to be located. The usage intensity of the considered facility is given by the sum of the currents entering such a node.

In alternative, as long as the facility has a limited capacity *Cj,* the node where the facility is placed is not directly put to mass, but its connection to the null potential level is that depicted

Figure 2: The electric equivalent of a capacitated facility located in the j^{th} node



V_j is equal to zero until $I_j=C_j$, that is up to the facility saturation. As soon as $I_j>C_j$ the diode becomes cut-off and V_j increases, practically re-distributing the clients among the other facilities, while keeping $I_j=C_j$ It is worth noting how this electrical effect resembles a waiting-in-queue time. More precisely, the queuing time of the j^{th} facility is modelled by the potential V_j.

3.4. The "quality index"

Up to now, we have assumed that the choice made by the clients to reach a certain facility, among those of the same type present on the territory, is mainly dictated by the time to access the facility: i.e. the user selects the facility which turns out to be "closer" in a temporal sense. As previously observed, there could be other factors strongly affecting the facility selection process. Among them, the crucial one is undoubtedly the quality level, which the facility can provide. If this factor is taken into account, it can results that the client is likely to accept a longer access time to reach a facility with a quality level higher than the average one. The point is to quantify in a "equivalent access time" measurement the concept of "facility quality level", i.e., to associated with each facility a suitable "quality index".

Quality depends on a number of elements, some of quantitative nature, other of qualitative nature. Thus, to transform in a unique quantitative measurement this miscellany of different elements, a way could be that of following the lines of fuzzy logic to come up with a "crisp" value expressing the "equivalent access time" of the facility, to be combined with the actual access time.

3.5. Solution of the network

Given the parameters Eij and Rij for any link of the electric network representing the transportation network as a function of the operating point and of the dependence of the travel time on the traffic intensity, specified, in each node, the magnitude of the current generators modelling the clients population, and, finally, chosen a feasible location of the considered facilities, then, solving the network means to determine:

1. The voltage at each node on the basis of which it is possible to quantify the access times, the border lines and the consequent partition of the network nodes which identifies the influence areas of each facility. Note that the facility access time is given by the difference between the potential of the source node, i.e., the starting point, and the destination node, namely the node where the facility is located.
2. The current in each link of the network which represents the traffic variation induced by the facility.
3. The current in the node where the facility is located which describes the degree of utilisation of the facility.

It is worth noting that the network solution, relying on the analogy between voltage at the nodes and travel times, and on the analogy between currents in the links and induced traffic, leads to the determination of the minimum of the cost function *C* indicated in (1). Indeed, in a circuit made up by resistors, independent voltage and current sources, the currents tend to reach a distribution such that the dissipated power is minimum.

4 ULISSE (Urban Location Interactive System for SErvices)

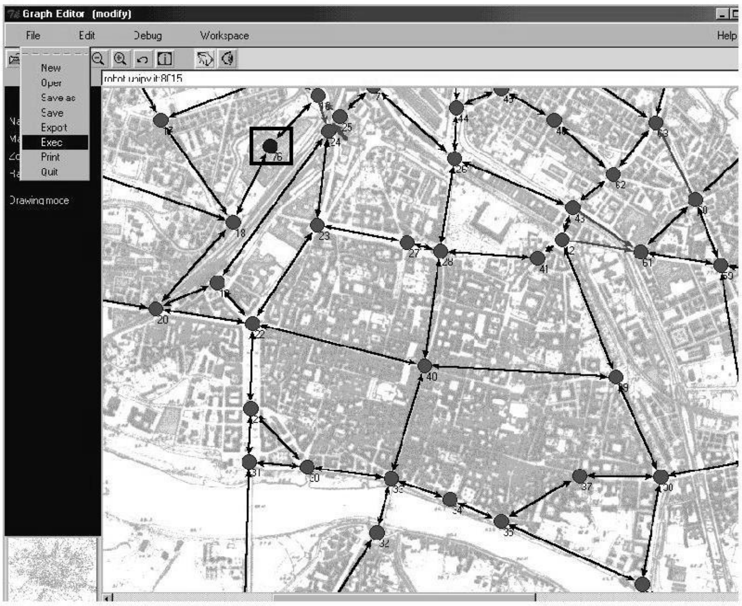
The main purpose of this software is to make the interaction with context data as direct as possible. To this end three kind of items are used: a map that gives the graphical foundation for the model graph; source nodes for population or crossroads and facility nodes for services; a number of links that connect existing nodes.

The program is base on different *rooms*, to draw the net, to calculate, to view the results. The user is asked to insert the considered map, and then to insert the network nodes and parameters. Each node and branch is characterised by special parameters as previously shown. The user can intervene every moment to edit each parameter in order to verify different possibilities and compare them.

IN Figure 3 is shown an ULISSE's window of the graphic interface shows a step of the creation of the net.

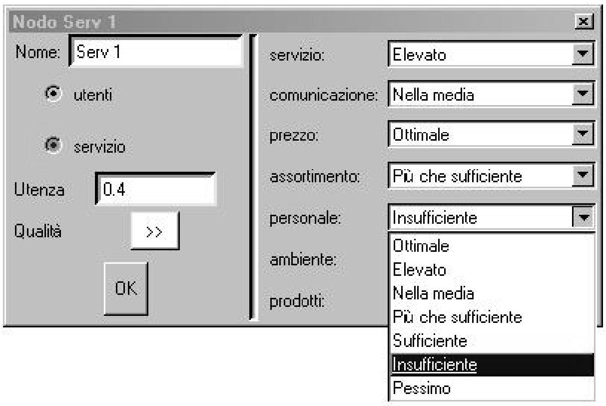
The commands are similar to the classical Windows ones, in order to have an immediate familiarity with the program, which is based on LINUX platform. Clicking on the icons can activate each command. The client nodes are coloured in red, the facilities in blue (in the picture is pointed out with the square).

Figure 3: ULISSE's window for creating a mobility net grounding of the map of Pavia



Each service can be characterised by a quality index; this value, expressed in seconds, is an input data for the planner when he knows exactly which kind of facility is treated (i.e. specific commercial services). In other cases it's useful to let clients estimate it; a common client is not often able to express in a precise number the quality of a facility; it can be estimated using the *fuzzy set* starting from common spoken terms. ULISSE offers the possibility to enter directly the quality value or, entering a set of characteristics of the services, to calculate automatically the equivalent access time using fuzzy models. Figure 4 shows the valuation window.

Figure 4: quality valuation with fuzzy parameters



4.1. Application

When inserting the nodes and branches, ULISSE creates automatically the calculation netlist that SPICE solves. The interface permits to have the view of the solution directly on the plan.

One of the most interesting and actual problems in urban planning is the planification of the dismissed areas. In the last years the urban planners have concluded that the best strategy to follow is to provide these areas with different urban functions (residential, offices, facilities) in order to have a 24 hours a day used urban spaces. The services have clearly a focus rule. The following examples concern the city of Pavia; it's analysed which is the best location of facilities choosing between the different dismissed areas of the city. In figure 5 is presented the considered mobility net, with the positioning of two facilities in two areas near the centre. The first example the two facility are considered with a quality value equal to 0, and in figure 6 is shown how the clients divides to reach the two services. In the second example the node on the western part is characterised by a quality value equal to 100 and it's clear how this connotation incises on the distribution of the users (figure 7).
For each example ULISSE can calculate and visualise the influence area for each node, run through time of each branch, the time to go from a client node to the selected node, the number of clients going through each branch, the total cost for people to use the services. The optimisation is interactive and the user (planner) can compare the different hypothesis foreseen, changing every parameter on line in real time.

Figure 5: Pavia's mobility network with two services

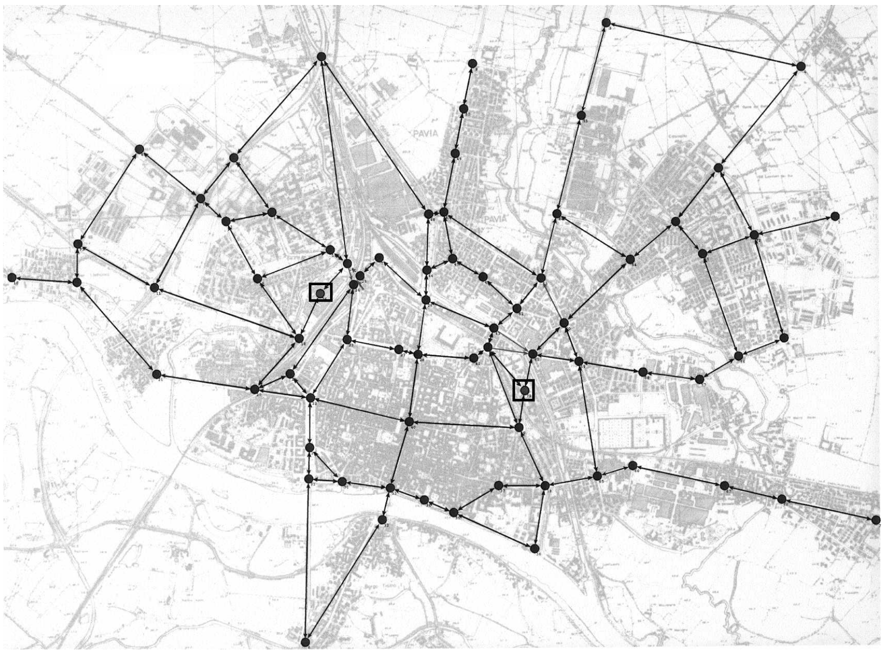


Figure 6: influence areas of the two services

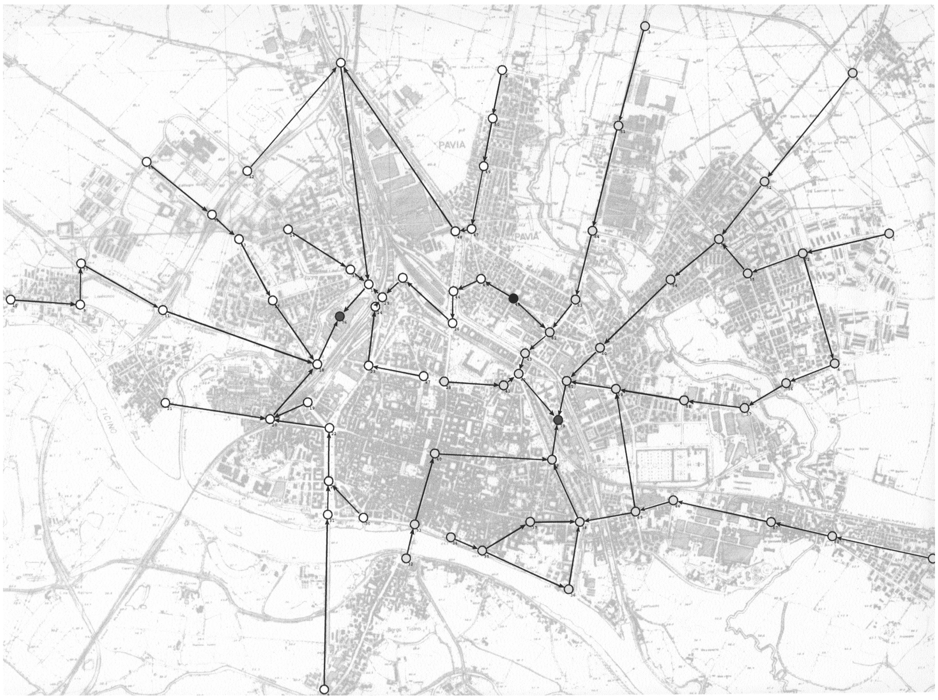
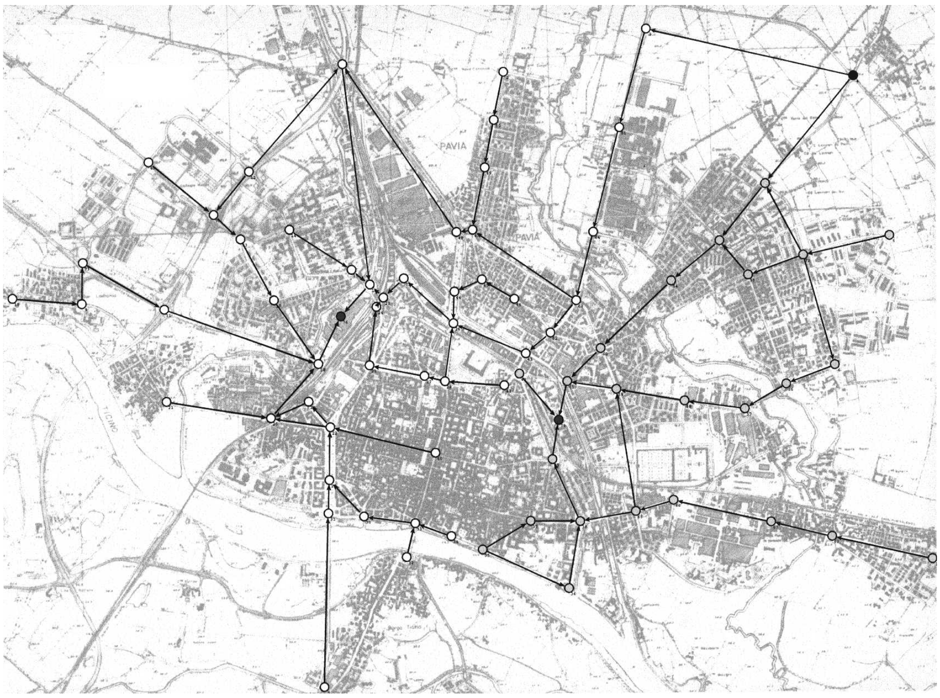


Figure 7: influence areas (the western service is “better” than the eastern)

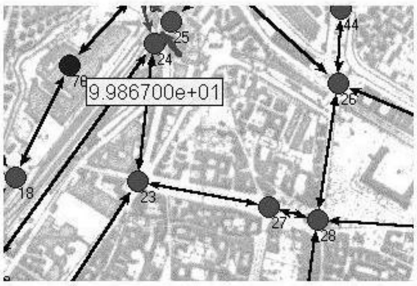


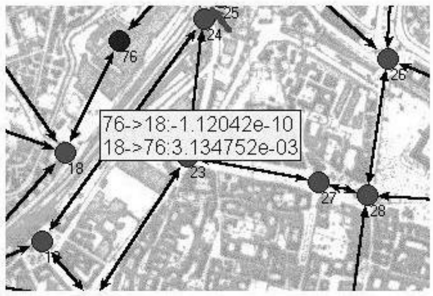
4.2. *Results*

The first grafic result (fig. 6) shows the influence area of each service; the city is divided in two equivalent parts. In the second case (fig. 7) the influence of the quality makes the western node attract more clients, moving the ideal basin border line to east in the northern part.
ULISSE can provide different kind of outputs: graphics and tables. Each case might be analysed singularly looking at particular set of data, depending on the specific analysis required. In the previous examples is underlined the interaction between the planner and the system: the planner can intervene on-line on the parameters having output in less than one second and compare the solutions reaching for the one he considers the best, in an iterative process. Moreover the comparison of the two different hypothesis suggests a lot of implications i.e. how much is possible to improve the service quality in order to obtain the best area's value without weighing too much on the global cost.

It's possible to have a visualisation of each calculated parameter (i.e. the run through time for each branch, the number of clients for each branch, the number of clients for each service, etc.) just going with the cursor near the node or branch considered.

Figure 8: visualisation of results





ULISSE could be used also for traffic planning problems, starting from the testing of new fast car ways or new public systems of mobility, to the consequences on the traffic due to the closing of some streets for upkeep works or for new pedonal areas. Besides for example a matrix *N x N* consisting of the minimum path for each couple of nodes i,j, 0<i,j<*N,* can be built. Each element n_{ij} is determined, through the proposed model, assuming to have a small number of clients entering only node i (the source of the path), a single facility located at node i (the destination of the path), and observing the links which are affected by the induced traffic Of course, the obtained minimum path is strongly dependent on the load condition of the network. To compute all the elements of the matrix the analysis is repeated *N* times, taking advantage of the intrinsic parallelism of electrical network analysis.
The most important characteristics of ULISSE are the continuos interaction with the user, the calculation time (which is always less than one second), the viewing of the result directly on the plan, the possibility to use different thematic plans and verify the impact on each singular layer of the choices planned.

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